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WILLIAM HUGGINS, Esq., F.R.S., President, in the Chair.

Andrew Ainslie Common, Esq., 37 Eaton Rise, Ealing; and
James Love, Esq., Talbot Lodge, Bickerton Road, Upper
Holloway,

were balloted for and duly elected Fellows of the Society.

Visibility of Mercury—and of Venus in Sunshine.

By William F. Denning, Esq., F.M.S.

(Communicated by the Rev. R. Main, M.A., F.R.S.)

The favourable east elongation of *Mercury* in May allowed him to be seen here with the naked eye on thirteen evenings; in fact, his detection between the 5th and 28th was merely a question of weather; he was, within these dates, visible every night on which the sky was clear enough. In the early part of the month (7th, 8th, and 9th) he looked brighter than a 1st mag. star on the horizon, being much plainer than *Mars* (about 13° E.), and comparing favourably in point of lustre with *Aurigæ*. On the 13th the planet was first seen at $8^{\text{h}} 20^{\text{m}}$, and remained in view until $9^{\text{h}} 40^{\text{m}}$, which gives a duration of visibility of $1^{\text{h}} 20^{\text{m}}$, and this was a fair average period for a clear night between about the 10th and 20th. On the 16th the planet was first glimpsed at $8^{\text{h}} 35^{\text{m}}$, and on the 18th (sky partly cloudy) at $8^{\text{h}} 41^{\text{m}}$, continuing visible till near setting, though on these dates he had become much fainter than on the 7th, 8th, and 9th. On the 19th I watched the planet from $8^{\text{h}} 32^{\text{m}}$ to $9^{\text{h}} 55^{\text{m}}$ ($1^{\text{h}} 23^{\text{m}}$), but his appearance was feeble, especially during the latter part, and certainly less than 2nd mag. On the 21st (day of extreme elong.) the air was very clear in WNW., and the

planet seen at 8^h 55^m, but very faintly; his light had evidently gone on decreasing rapidly since the earlier observations. Cloudy weather here supervened. On the 28th the sky was quite clear after sunset. I saw *Mercury* in the finder of my 10 $\frac{1}{4}$ -in. reflector at 8^h 52^m, and could just glimpse the planet with naked eye at 9^h 20^m, but it was one of the faintest objects I have ever seen, and smaller than a 3rd mag., which I could have found more easily in the same place. I saw β *Geminorum* (near *Venus*) as early as 8^h 35^m. The planet set 1^h 50^m after the Sun on this date, and for some days afterwards continued well situated as regards elongation, but was totally beyond reach of the naked eye; and the actual decrease in brilliancy between the 7th and 28th must have amounted to about 3.5 mag. The stronger twilight prevailing during the latter part of the elongation partly accounts for his extreme feebleness then, and the difference of phase (from gibbous to a crescent) had a good deal to do with it, though the semi-diameter had increased from 2''.9 on the 5th to 4''.4 on the 25th. The loss of phase, however, was not nearly compensated for by increase of size; and this is pointed out in Lardner and Dunkin's "Handbook," in which it is stated of *Mercury* that "it is brightest at a less [than max.] elongation, even though it moves to a greater distance from the earth, since it gains more by the increase of its phase than it loses by increased distance and diminished elongation." This refers to the planet when nearing superior conjunction at W. elongation, but it is also true, conversely, of the E. elongation; and it would appear, therefore, that he is most favourably visible as a naked-eye object a few mornings *after* max. elong. W., and a few evenings *before* max. elong. E. This fact has been strongly marked in previous years, notably in February 1868, when he appeared to rival *Jupiter* on several nights preceding extreme elongation, and on the evenings of May 31, June 1, 2, 3, and 4, 1875, the planet was seen as a conspicuous object though the maximum elongation was not attained until June 10. This may account for the many futile efforts to see *Mercury*. Elongation is a necessary element in his detection, but a crescented phase may so enfeeble his light as to render him utterly beyond reach, as on May 29th.

Venus has been quite conspicuous as a naked-eye object in sunshine during the last three months. The first time I glimpsed the planet (during the present evening apparition) was on February 22, at 4^h 45^m. On February 23 she was seen at 1^h 45^m, and on February 28 precisely at noon. On the latter date I watched the planet for about 30^m; she was somewhat faint, though I found her readily enough a few degrees west of the crescent of the new moon. She was plainer on March 4 at 1^h 40^m. The planet must be rarely seen at noon so early as February 28, as the circumstances are unfavourable. On the occasion now referred to the semi-diameter was scarcely 7'', and altitude 35°. The fact of her visibility at such a time proves with what ease she may be seen under better conditions.

During the present month her high altitude has rendered her surprisingly bright as a daylight object, and in strong contrast with her feebleness when I saw her at the end of February.

Bristol, May 30, 1876.

On the Atmosphere of Venus.

By E. Neison, Esq.

In the *Astronomische Nachrichten* for 1849 (No. 679, vol. xxix. p. 107) are published some observations, by Clausen and Mädler, of the prolongation of the cusps of *Venus* when the planet was near conjunction, and which showed that the Sun's rays underwent a considerable refraction from the atmosphere of the planet. The extent of the prolongation of the cusps of *Venus* was measured with the great Dorpat telescope, and from these measures the amount of the horizontal refraction exerted by the atmosphere of *Venus* was ascertained by means of formula which was given.

This formula was as follows:—

Let V = the angular distance between the centres of the Sun and *Venus*;

x = the prolongation of the cusps of the planet;

S = the semi-diameter of the Sun;

ϕ_1 = the radius-vector of *Venus*.

Then the horizontal refraction ζ_o of the atmosphere of *Venus* was found from the equation

$$\zeta_o = \frac{1}{2} \left(\psi - \frac{S}{\phi_1} \right),$$

where

$$\sin \psi = \sin V \sin x.$$

By this method Mädler found the horizontal refraction of the atmosphere of *Venus* to be 43'.7, or about $\frac{1}{6}$ th greater than the Earth's. This result Mädler subsequently sent to the Astronomer Royal, who communicated an account of the same to the Royal Astronomical Society, and it appeared, in due course, in the *Monthly Notices* (vol. xviii. p. 320).

Lately this formula has been employed by Professor C. S. Lyman, of Sheffield, United States, to reduce similar measures of the prolongation of the cusps of *Venus*, made with a fine 9-in. refractor during 1866 and 1874. By its means Professor Lyman found from his observations 45'.3 and 44'.5 for the horizontal refraction of the atmosphere of *Venus*.

A note by Mr. R. A. Proctor, in the *Astronomical Register* (October 1875), induced me, as soon as I had the leisure, to

examine this formula of Mädler's, when it was at once apparent that as it had been employed by Mädler and Lyman it was defective. Instead of the angle V or the angular distance between the centres of the Sun and *Venus* as seen from the Earth which they had used, they should have employed the supplement of the angle between the Sun and Earth as seen from *Venus*. In consequence of this error the value which they have deduced from their observations for the horizontal refraction of the atmosphere of *Venus* is incorrect.

Retaining the notation above, and for additional accuracy putting

S' = the semi-diameter of *Venus*,

ϕ = the radius-vector of the Earth,

then the proper equation for finding the horizontal refraction is

$$\zeta_0 = \frac{\phi}{2\phi_1} \{(V + S' \sin x) \sin x - S\}.$$

The above expression is easily found, for if in the triangle formed by the Sun, Earth, and extremity of the prolongation of the cusp of *Venus*, we put θ for the angle at the Earth, and if ψ represents the apparent deflection of the solar rays, or the supplement of the angle at *Venus*, then

$$\frac{\sin \psi}{\phi} = \frac{\sin \theta}{\phi_1}.$$

It is easy to see that from this definition we have

$$\theta = V + S' \sin x,$$

and

$$\psi \sin x = 2\zeta_0 + \frac{\phi}{\phi_1} S.$$

It only remains to replace the ratio of the sines by the ratio of the angles, which is permissible, the angles being very small, and the above equation results at once.

Applying this formula to the observations given by Mädler in the *Astronomische Nachrichten* and the values for the horizontal refraction, which results are

60'12, 54'33, 47'70, 49'72, 49'97, 58'73, and 60'44;

the mean value is 54'43, or 54'26'', and a probable error of 1'38.

The four observations by Lyman, made during 1874, are probably far superior in accuracy to Mädler's, and were made under better conditions. They are each the result of a number of repeated measures, and give for the horizontal refraction the values

56'34, 51'54, 54'63, and 51'49;

the mean value is $53'50$ or $53'30''$, agreeing closely with the result obtained by Mädler.

The mean between these two results is $54'65$ or $54'39''$, and this value is probably within one minute of arc of the true value of the horizontal refraction of the atmosphere of *Venus*. The details of the measures made by Lyman in 1866 not having been published, it has not been possible to reduce these; considering, however, the nature of the error in the formula by which they were reduced by Lyman, the mean of these observations cannot differ much from the value $54'7$, a result well in accord with the other two series.

It appears, therefore, that the value found from their observations by Mädler and Lyman, for the horizontal refraction of the atmosphere of *Venus*, must be increased from $44'5$ to $54'65$.

From the value of the horizontal refraction of the atmosphere it is easy to deduce the surface density of the same. The method of effecting this has been already given (*Monthly Notices*, vol. xxxv., p. 331). Supposing the surface density equal to that of the Earth's, the horizontal refraction which it would exert could not differ much from $28'53''$. Consequently, the value found above for the amount of the horizontal refraction shows that the density at the surface of *Venus* of its atmosphere cannot differ much from being 1·892 times that of the Earth's, or nearly twice as dense. This is entirely in accord with the phenomenon presented by the planet, which indicates the presence of an atmosphere materially denser than that of the Earth's.

Repeated observations have shown that when *Venus* is near the Sun the entire planet can be seen as a dark circular disk. This appearance may arise either from the planet being seen projected against a bright background—such as, for example, the solar corona—or else from being surrounded by a faint ring of light due to the refraction of the solar rays by the atmosphere. Reversing the formulæ already given, it is easily found that the maximum distance from the Sun at which the planet's atmosphere has power to refract the solar rays so as to form a complete ring of light around the planet is

$$V = 2 \frac{\phi^1}{\phi} \zeta_0 + (S - S').$$

The greatest value this can have is $1^\circ 37'$. But the ring of light would be sufficiently near completeness at a distance of even 2° to present the same appearance. From the natural tendency of the eye to supply the missing portion of a faint ring of light, it is very probable that, except in very careful observations, even at a distance of 5° from the Sun, the prolongation of the cusps of the planet (about 20°) would be sufficient to make it present the appearance of a circular dark disk on a brighter background.

Observations of Venus. By Capt. W. Noble.

Having been in the habit, for many years past, of observing *Venus* in nearly every part of her orbit, with a very considerable range of powers, and under the most varying conditions of atmosphere and illumination; I was very much struck by Mr. Brett's announcement, at our last meeting, that he had detected unmistakable specular reflection from the body of that planet. Believing that, had this phenomenon been anything like so apparent as his very artistic drawing represented it to be, I *must* have seen it, I yet so far mistrusted my own considerable experience, as to admit the possibility that Mr. Brett's exquisitely-trained eye might have detected an appearance which had been previously overlooked by others less specially employed in regarding form, light, and shade, with a view to their reproduction. Since the 12th of last May, therefore, I have availed myself of every favourable opportunity to investigate this question of alleged specular reflection from *Venus*; but, I may as well say at once, with the most absolutely negative results.

My first step was to examine the planet, which I did in daylight, twilight, and at dusk, with various powers from 84 to 394, to see whether I could detect any indications of specular reflection by ordinary telescopic vision. I could discover none whatever; the whole of the light in the neighbourhood of the illuminated limb was as homogeneous as it possibly could be. I then, using a power of 255, interposed a graduated shade of dark glass; traversing it, with the utmost slowness, before the eyelens of my eye-piece until the planet was rendered invisible. Under these circumstances the first part to fade away was (as might, *à priori*, have been expected) that in the neighbourhood of the terminator: and the effect presented was, of course, that of the material reduction of the breadth of the crescent. Then the actual cusps vanished; and, finally, the central part of the illuminated periphery of *Venus* went out altogether; not the slightest sign, trace, or indication of specular reflection—or of one part being in the smallest degree brighter than another—being visible. With a very dark green eye-glass too, of uniform tint, which extinguished the cusps, the light of the limb remained absolutely without difference of brightness of any description.

In addition to these observations, made and repeated on numerous occasions with my 4.2-in. Ross achromatic, I have examined the planet with the beautiful 6.25-in. Browning—with reflector belonging to my friend Mr. H. J. Slack, F.G.S., &c., employing his own wedge of dark glass for cutting off the light of the planet—and with precisely the same result. I can then only come to the conclusion that *Venus* does *not* exhibit anything whatever in the nature of specular reflection; and that its apparition, in Mr. Brett's case, must have had its origin either in his instrument or in his eye.

It may not be uninteresting to add that, while my experiments were in progress, I discovered that, by waiting until the evening was dark enough, I could, with a power of 154 (or, in fact, any other—but this is the one with which I make the discovery) distinctly make a quasi-phosphorescent body of *Venus* out of the blue light resulting from the very slight over-correction of my object-glass. This “dark limb” was quite startling in its apparent objectivity; but, as pointed out by Mr. Green in his oral address at our April meeting, it vanished completely on hiding the illuminated crescent.

*Forest Lodge, Maresfield,
1876, May 27.*

Photometric Experiments upon the Light of Venus.

By John I. Plummer, Esq.

The planets *Venus* and *Jupiter*, it is well known, are frequently sufficiently bright when favourably situated to cast a well-defined shadow, but hitherto advantage has not been taken of this circumstance to determine the relative intensities of their light to that of the Full Moon, nor to compare it with any standard terrestrial source of light. The following experiments and observations have been attempted with a twofold purpose: First, to ascertain whether this troublesome problem might not be attacked successfully from a new point of view, so that estimates of relative brilliancy may be made in the simple manner I have to describe as reliable as those made with costly photometers; and secondly, to determine by observation alone the variations in the amount of light reflected from *Venus* towards the Earth at its various phases. As photometric comparisons are known to be liable to much error, and are necessarily affected by the peculiarities of vision of the observer, it may be thought desirable that they should be made not only in as many different ways, but by as many different persons as possible. This is my excuse for attempting a problem various portions of which have received the attention of Herschel, Bond, Zöllner, and still more particularly of Seidel. Moreover, *Venus*, owing to its close proximity to the Sun, has had less attention paid to it than any other, while its brightness is such that it is most suitable for the direct comparison of shadows.

The plan that I have adopted has been to compare the light of the planet with that of a standard sperm candle burning 120 grains of wax per hour, and to vary the distance of this until the shadow it casts upon a screen of white paper has an equal degree of intensity to that given by the planet. It being found impossible to get the two shadows upon the same screen, separate screens were arranged for each, and brought as near to each other as possible. The arrangement was therefore a modification of Rumford's photometer. The objects of which the shadows were

observed were two equal cylindrical steel wires of $\frac{1}{14}$ -inch diameter, placed in a dark room nine feet in front their respective screens; but the judgment was further assisted by noting also the shadows of the wooden laths to which the wires were attached, and of which the thickness was about $\frac{1}{2}$ -inch. These conditions were preserved throughout the whole of the observations. To protect the candle from wind, it being necessarily placed in the open air, it was fixed within a lantern, which was itself enclosed, except upon one side, in a rough wooden box painted a dull black. These precautions are believed to have been sufficient for their purpose.

That in dealing with the light of *Venus* and the shadow which it is capable of throwing upon a screen properly situated we have not to deal with a faint or doubtful trace, the following facts may be noted: Thus on March 29, the Moon being four days old and within 12° of *Venus*, the light from each was received upon the same screen, and the shadow from *Venus*-light was still conspicuously visible. Upon the same evening *Sirius*, though having a light-intensity about $\frac{1}{3}$ th that of *Venus*, was also observed to cast an appreciable shadow, the direct light of the Moon being, of course, excluded from the room in this case. That *Sirius* is sufficiently bright to produce a definite illumination upon a screen has not, I believe, been previously observed; and I may mention here that there would seem to be no difficulty in determining by this method the relative brilliancy of those planets which exceed *Sirius* in brightness; and I have been deterred from at present extending this investigation to *Jupiter* and *Mars* only by the impossibility of obtaining a sufficient range through which to vary the distance of the candle. On April 26, the light of *Venus* and of the young Moon were observed to fall upon a plaster wall in the open air, and the shadows of neighbouring buildings referable to each of these bodies were distinctly observable at the same time. For obvious reasons, no observations have been taken when the Moon was near full; and only on one occasion an observation was taken when the Moon was above the horizon; but there would not have been the least difficulty in continuing the observations under all circumstances. The following comparisons have been made:—

	h m	Observed Distance of Candle. Feet.	Calculated Altitude of φ °	Log C =		Phase of φ °
				Log $\frac{C}{C_{\text{corr. for extinc.}}}$ Candle at 1 ft. dist.	Log C $\frac{r^2 \Delta^2}{r_0^2 \Delta_0^2}$	
1876.						
Mar. 20	8 40	290.65	12 42	5.1715	5.2198	114.9
" 24	8 44	334.5	13 31	5.0314	5.0561	113.1
Apr. 14	8 52	286.25	19 41	5.0636	4.9458	102.5
" 21	8 59	263.75	20 20	5.1269	4.9537	98.9
" 23	9 6	265.0	19 38	5.1322	4.9423	97.9
" 26	9 22	306.0	17 50	[5.0311]	[4.8153]	96.4
May 11	9 46	275.42	15 30	5.1605	4.8013	84.1
" 13	10 26	314.19	6 37	[5.3623]	[4.9817]	83.2

Upon all these occasions the sky has been brilliantly clear and the equality of the shadows has been considered satisfactory by at least two persons. The probable error of a single determination is believed not to exceed 6 feet. The correction for extinction of light by the atmosphere has been obtained from Seidel's Table, but in order to be as independent as possible of arbitrary constants those corrections only have been applied to reduce the results to an uniform altitude of 18° , which is nearly the mean of the observations. Rejecting the results obtained on April 26 and May 13 (the first because the Moon was shining at the time of comparison, and the second because *Venus* was very low and appears to be over-corrected for extinction), and taking the means of results on March 20, 24, and April 14, 21, 23, we obtain the ratios $\frac{1}{78152.5}$, $\frac{1}{77861.0}$, $\frac{1}{69101.9}$, to express the re-

lation of *Venus* light to that of the candle at the distance of one foot when the illuminated portion of the planet is 114° , 100° , and 84° respectively, showing that *Venus* approaches its point of maximum brilliancy on June 6 very gradually indeed. Or taking the mean of the six comparisons, we have the light of *Venus* when 101.9 of its illuminated disk is turned towards us and its altitude is 18° , equivalent to that of a sperm candle at the distance of 276.3 feet and its intensity equal to $\frac{1}{276.3^2} = \frac{1}{76343}$ part of that of the candle at the distance one foot.

The next step was to reduce the values obtained (Column IV.) to some assumed distance of *Venus* from the Sun and Earth, and to exhibit the comparison of light intensities affected only by the changing phase. The distances assumed for this purpose r_0 and Δ_0 are 0.7184 and 1.0000 respectively, and the results are given in column V. I have then compared, by means of a graphical construction (see Diagram), the curve that best satisfies the observations with those yielded by both Lambert's and Euler's formulæ, taking the intensity of light when $v = 84.1$ as unity; and, although the interval of time over which the observations extend is too limited to admit of a definite conclusion being drawn, they appear to indicate that *Venus*, like the Moon, increases in brightness as it approaches the full phase much more rapidly than can be accounted for upon theoretical considerations only. It will be seen also that Lambert's formula much more nearly satisfies the observations than does Euler's. Some further evidence on this point may possibly be obtained after the inferior conjunction, when the twilight being of shorter duration will interfere less with the observations.

To obtain results independent of any terrestrial source of light it was necessary to compare the sperm candle that had been employed in the observations of *Venus* with the Full Moon, and to do this it was found more convenient to have recourse to the principle of Bunsen's photometer. The comparisons were effected upon April 8 and May 7, and they have

been treated in precisely the same way as those of *Venus*, except that in addition a small correction has been applied to reduce the light of the Moon on May 7 to the full phase. No such correction is needed on April 8. Euler's formula $\left(\theta = \sin^2 \frac{1}{2} v \right)$ has been used for this purpose. The results are tolerably accordant, giving the light of the Full Moon at the altitude of 18° , its semi-diameter being $934''\cdot67$, equivalent to that of the candle at distances of $8^{\text{ft}}\cdot888$ and $8^{\text{ft}}\cdot641$, or giving weight, according to the number of comparisons on each evening, of $8^{\text{ft}}\cdot722$. Combining this value with that of the mean of the observations of *Venus*, we have,

$$\frac{\text{Actual Light of Venus } (v = 101^\circ\cdot9)}{\text{Mean Full Moon}} = \frac{1}{1004}.$$

A more accurate mode of treatment would probably be to reduce each observation separately to some standard phase, and take the mean of the results. Assuming, as Seidel has done, that *Venus* attains its greatest brilliancy when $v = 76^\circ\cdot13'\cdot5$, $\Delta = 0^\circ\cdot53944$, and $r = 0^\circ\cdot7233$, and using Lambert's formula

$$\left(\theta = \frac{\sin v - v \cos v}{\sin v_0 - v_0 \cos v_0} \right)$$

to effect the reduction we obtain

$$\frac{\text{Venus at greatest brilliancy (alt. } 18^\circ)}{\text{Standard sperm candle at 1 foot dist.}} = \frac{1}{60813\cdot8} = \frac{1}{246\cdot62}.$$

If the suspicion to which I have alluded be correct, that Lambert's formula is not strictly applicable to *Venus*, this result will, of course, be slightly in excess of the truth. Again, introducing the above-determined value of the mean Full Moon, we have

$$\frac{\text{Venus at greatest brilliancy}}{\text{Mean Full Moon}} = \frac{1}{799\cdot5}.$$

I have not been able to find any previous comparison of *Venus* with the Full Moon with which to compare this result, except what may be inferred from Bond's observations (*Memoirs of the American Academy*, vol. viii.), which give the light of *Jupiter* at mean opposition equal to $\frac{1}{6430}$ that of the mean Full Moon, and the ratio of the light of *Venus* at greatest brilliancy to *Jupiter* as $4\cdot864$ to 1, which differs only very slightly from Seidel's comparison of these two planets. This would yield the ratio $\frac{1}{1322}$ as comparable with my result; or, in other words, I attribute to *Venus* a brilliancy about 65 per cent. greater than

Bond has found. Since the methods employed are entirely dissimilar, and since Bond's investigation has chiefly to do with the Moon and *Jupiter*, both of which he has observed at altitudes generally much greater than those at which the observation of *Venus* is possible, this discordance does not prove much.

Orwell Park Observatory, near Ipswich,
1876, June 1.

On the Proper Motion of Bright Spots on Jupiter.

By John Brett, Esq.

There have recently appeared upon the disk of *Jupiter* two bright spots, of such distinctness and isolation as peculiarly adapt them for measurement, with a view to determine either the rotation of the planet, or their own proper motion.

I beg leave to lay before the Society the record of four observations of these spots, forming a series extending over a period of 286 hours 20 minutes of mean time, and to ask whether they afford evidence of proper motion, or whether, on the other hand, they tend to cast any doubt on the accepted rotation of the planet. There were several peculiarities about these two spots which seem to me to give them an eminent claim to attention. They occurred very near to the equator, and were very well defined, and free from entanglement with other markings: an advantage which they have maintained with singular uniformity throughout the period mentioned; but the special peculiarity to which attention is asked is, that during an interval of five days they remained in the same relative position, without any variation whatever.

Their stability in respect of latitude during those five days is undoubted; but the question is, whether or not they were equally stable in longitude. This remark only applies to the first five days of the series, because at the end of twelve days a certain deviation was obvious, as may be seen from the drawings which accompany this paper.

The drawings are fairly accurate, and I have no excuse to offer for them if they are not so. It is more easy for a practised draughtsman to detect differences of position or extension by the unassisted eye than by the micrometer-wire; and this is especially true of such delicate phenomena as planetary markings.

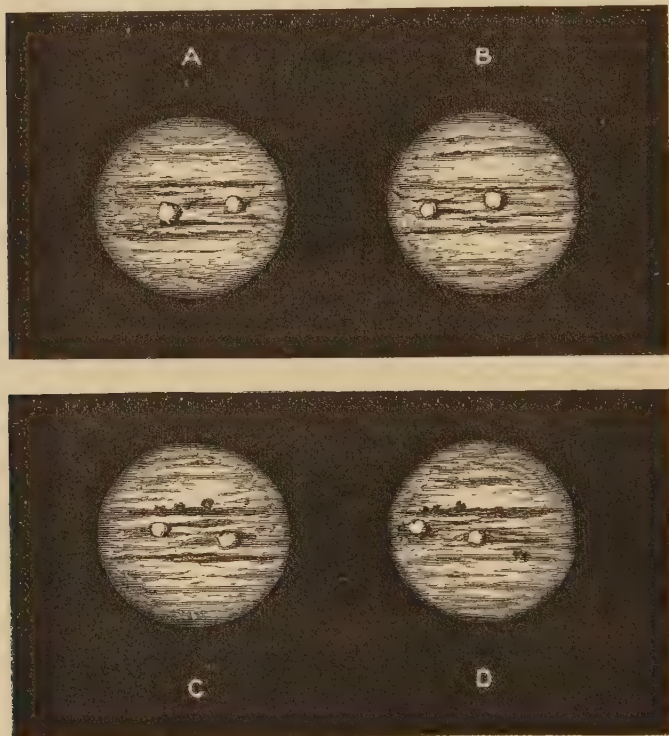
The first observation was made on May 23rd, 1876, at 11^h 30^m G.M.T., and is recorded by the drawing marked A.

The second observation, May 28, 1876, at 10^h 30^m G.M.T., by the drawing B.

The distance between the two spots occupies about 42° of Jovian longitude, or about 33,000 miles. Their diameter is nearly

equal, being estimated at $\frac{1}{14}$ of the planet's diameter, or 6,310 miles. The interval of time between these two observations is 119 hours, that is to say, 12 rotations of the planet, according to Airy's determination, during which time their distance apart and their latitude remained constant. The remark to be made upon this pair of observations is that there is a discrepancy between them amounting to $44^m 6^s$ of time.

Assuming Airy's rotation, viz., $9^h 55^m 21^s$, the spots have gained on the planet's surface at the rate of $4^m 2^s$ in each revolution.



Between the latter of these two observations and the next of the series there is an interval of seven days, = 17 rotations of the planet; and the same two spots turn up again somewhat earlier than the calculated time.

It unfortunately happens that on this occasion their configuration has undergone some change; but their dimensions and the distance between them remain very much as before. The most important circumstance respecting them is, that their rate of progress shows a certain acceleration.

They appear to have gained on the assumed rotation during this interval at the rate of $7^m 14^s$ per revolution.

Between the drawings C and D, the last of the series, there is an interval of 20 minutes of time, during which the spots passed over about 15° of longitude, proving that they were certainly not losing on the rotation.

Taking the series of observations together, the conclusion I derive from them is that the period of the spots is less than the rotation of the planet, and that their least proper motion is 165 miles per hour.

It will not be denied that this, if established, is a very remarkable circumstance.

The only alternative inference, viz., that there is an error in the assumed rotation, is negatived by the consideration that the rate of the spots is not uniform. It must, however, be remembered that the rotation is derived from observations of some such spots.

The proper motion theory is not without its difficulties. It must not be forgotten that these phenomena which are described as bright spots are bodies of approximately globular form. The fact of their casting shadows was discussed before this Society two years ago, and the existence of those shadows has since been confirmed in a paper presented to the French Academy last November by M. Faye.

The shadows of the two spots under consideration are sufficiently pronounced to indicate a globular form, and to afford evidence that light can penetrate the planet to the depth of more than 6,000 miles; but that the material of which it consists is sufficiently limpid at that depth to allow bodies nearly as large as the *Earth* to swim about in it seems at first sight rather startling, not to mention the question what inducement or excuse they could have for advancing in the direction of the planet's rotation.

The fact that they are wholly immersed in the semi-transparent material of the planet is indisputable, since they gradually disappear as they approach the limb, and in no case have been seen to protrude beyond it.

I cannot imagine any subject in physical astronomy on which diligent observation is more required, or one on which good observing faculties may be more fruitfully employed. I may mention that a 9-inch reflector shows these phenomena in a very satisfactory manner, but they *can* be seen with a 4-inch refractor. As the planet approaches quadrature the shadows ought to become more obvious, so that we may hope to hear more of them before the next session of the Society.

June 9, 1876.

On a hitherto unnoticed Apparent Inequality in the Longitude of the Moon. By Professor Simon Newcomb.

This inequality was brought to light in the course of an investigation having for its object the determination of the corrections to be applied to the lunar Ephemeris derived from Hansen's *Tables de la Lune*, in order that that Ephemeris might be used for the determination of such of the longitudes of Transit of *Venus* stations as depend on observations of occultations or Moon culminations. The material used was the series of Greenwich and Washington meridian observations of the Moon, extending from 1862 to 1874, and the residual errors of the Moon's place from Hansen's Tables thence derived. These residuals were first corrected for the theoretical terms in which the tables seem to need emendation, and for error of Moon's semi-diameter, and then for the mysterious error of epoch which has been so rapidly developed during the past ten years. The corrections thus applied, except the last, using Hansen's notation, and putting for brevity

$$D = g - g' + \omega - \omega',$$

which is equal to the mean longitude of the Moon, minus that of the Sun, were as follow :—

	"
For error of semi-diameter	+ 1'00
For correction of parallactic terms	+ 0'96 sin D
	- 0'13 sin (D + g')
.. annual equation	+ 0'09 sin (g')
.. accidental error in tables	- 0'62 sin (2D - 2g' - 2\omega').

Hansen's increase of his inequalities by the constant factor .00015, on account of supposed want of coincidence between the centre of gravity and centre of figure of the Moon, was not removed from any of the terms except the annual equation, because reasons are shown why both the evection and the variation should seem larger from meridian observations than from Hansen's theory.

When these several corrections were applied to the residuals, I was surprised to find systematic errors outstanding which could not be represented by corrections to the lunar elements. Of their reality there could be no serious doubt, because the Greenwich and Washington observations agreed in showing them. At first I was disposed to attribute them to inequalities in the surface of the Moon, but a more careful examination showed that they were periodic, and developed on the Moon's longitude, being at first positive when the longitude was between 180° and 360°, and negative in the first semi-circumference. But this was found to be subject to a sensible alteration, the point of

maximum positive error moving forward to about 0° and that of negative error to 180° in the course of a few years' observations.

It was now evident that, in the investigation of this inequality, its period might, during any one year, be regarded as the same with that of the Moon's mean anomaly, and that thus, during this interval, it would be identified with the correction to the eccentricity and perigee. Owing, however, to the want of exact coincidence of period, the apparent errors of eccentricity and perigee would be different from year to year—these elements would, in fact, appear to be affected with an inequality of long period.

In order to make the investigation more complete I also determined the corresponding corrections for the years 1847 to 1858, for which the residual errors of Hansen's Tables are given in the Greenwich observations for 1859. The resulting corrections and the mode in which the new term is brought out are shown in the following table. The residual errors of Moon's longitude during each term of about a year were supposed to be represented in the form

$$\Delta \delta l + h \sin g + k \cos g,$$

g being the mean anomaly, and h and k the coefficients to be determined from observation, on which depends the correction to eccentricity and perigee. Their values in terms of these last are—

$$h = -2\delta e$$

$$k = 2e\delta\pi.$$

The values of h and k , as they result immediately from observations, are shown in the second and third columns of the following table. Their periodicity is obvious. To make its law more evident, we find by a preliminary treatment that the constant portions of these quantities are approximately—

$$\text{const. of } h = h_0 = +0.50,$$

$$\text{const. of } k = k_0 = +0.10.$$

Subtracting these portions, we obtain the values of the purely periodic portions, which we call h' and k' , and which we present in the next two columns. We now see from these columns that in addition to h' and k' being separately periodic, they have each about the same period, and if one is represented by a cosine, the other will be nearly represented by the sine of the same angle. Let us then determine α and N from the equations

$$\alpha \sin N = -h',$$

$$\alpha \cos N = k'.$$

The values of α and of N thus deduced are shown in the two

following columns. The regular progression of the angle N is too striking to be overlooked, extending as it does through an interval of 27 years. We wish to see how it may be represented by a quantity increasing uniformly with the time. We first remark that the greater the value of a the more accurately will it be fixed by the observations. On this principle, we assign to N the weight in the column next following. The angle which best represents it, determined by least squares, is given in the column $\mu + nt$. I afterwards concluded that too great weight had been given to the older observations, and finally adopted, as the most probable value of N ,

$$N = 163^{\circ}.2 + 21^{\circ}.6 (t - 1868.5),$$

the residuals from which are given in the last column. The mean values of a are—

$$\begin{aligned} & \text{from the old series, } a = 1''.66; \\ & \text{from the new series, } a = 1''.55. \end{aligned}$$

The effect of the accidental errors of observation is to increase the value of a . I therefore consider that we may put—

$$a = 1''.50.$$

[It may be remarked that the values of a and N derived rigorously by least squares from the second series of observations above are—

$$\begin{aligned} a &= 1''.52, \\ N &= 161^{\circ}.2 + 22^{\circ}.8 (t - 1868.5)]. \end{aligned}$$

The table referred to is :

Mean Date.	h	k	h'	k'	a	N	Wt.	$\mu + nt$	ΔN	$\Delta' N$
	"	"	"	"	"	"	"	"	"	"
1847.8	-0.08	+0.55	-0.58	+0.45	0.74	52	1	94	+ 42	+ 24
48.9	-0.55	-1.38	-1.05	-1.48	1.82	145	3	118	- 2	- 44
50.1	-0.20	-1.91	-0.70	-2.01	2.13	161	3	141	- 20	- 36
51.2	-0.32	-1.92	-0.82	-2.02	2.18	158	3	165	+ 7	- 8
52.4	+0.26	-2.45	-0.24	-2.55	2.56	175	4	189	+ 14	0
53.5	+1.10	-1.88	+0.60	-1.98	2.07	197	3	212	+ 15	+ 2
54.6	+1.45	-1.40	+0.95	-1.50	1.77	212	3	236	+ 24	+ 12
55.8	+0.77	+0.31	+0.27	+0.21	0.34	308	$\frac{1}{2}$	260	- 48	- 60
56.9	+1.76	+1.82	+1.26	+1.72	2.13	328	3	284	- 44	- 55
1858.1	-0.17	+0.66	-0.67	+0.56	0.88	50	1	307	-103	-112

Mean Date.	h	k	h'	k'	α	N	Wt.	$\mu+nt$	ΔN	$\Delta'N$
"	"	"	"	"	"	"	"	"	"	"
1862.5	+0.04	+1.23	-0.46	+1.13	1.22	22	3	40	+ 18	+ 12
63.5	-0.64	+1.78	-1.14	+1.68	2.03	34	5	61	+ 27	+ 21
64.5	-1.07	+1.09	-1.57	+0.99	1.85	58	5	81	+ 23	+ 19
65.5	-1.03	-0.15	-1.53	-0.25	1.55	99	4	102	+ 3	- 1
66.5	-0.47	+0.10	-0.97	0.00	0.97	90	2	123	+ 33	+ 30
67.5	-1.93	-0.36	-1.43	-0.46	1.50	108	4	144	+ 36	+ 34
68.5	+0.34	-1.46	-0.16	-1.56	1.57	174	4	165	- 9	- 11
69.5	+1.67	-1.56	+1.17	-1.66	2.03	215	5	185	- 30	- 30
70.5	+1.48	-1.14	+0.98	-1.24	1.58	218	5	206	- 12	- 12
71.5	+1.65	-0.36	+1.15	-0.46	1.24	248	3	227	- 21	- 20
72.5	+2.15	-0.12	+1.65	-0.22	1.66	262	4	248	- 14	- 12
73.5	+1.91	+0.16	+1.41	+0.06	1.41	272	4	269	- 3	- 1
1874.5	+1.92	+0.60	+1.42	+0.50	1.50	289	4	289	0	+ 4

The corrections to the Moon's longitude arising from this term is—

$$\begin{aligned}
 \delta v &= -h' \sin g - k' \cos v \\
 &= -1''.50 (\sin N \sin g - \cos N \cos g) \\
 &= -1''.50 \sin (g + N - 90^\circ),
 \end{aligned}$$

which is the hitherto unnoticed apparent inequality of the Moon's longitude.

The period of N is $16\frac{2}{3}$ years, with a probable error of about half a year, and a possible error of a year or more. The corresponding period of $N+g$, or of the inequality in longitude, is—

$$27.4304 \text{ days} \pm 0.0040.$$

There is a large preponderance of probabilities against the real period being either less than $27^d.42$ or more than $27^d.44$. No known term in the Moon's longitude falls between these limits. The Moon's sidereal period is $27^d.32$, and her anomalistic period is $27^d.55$, so that the new term falls half-way between these two.

The non-accordance of this period with that of any term heretofore sought for is the reason why this term has not before been noticed. A term of unknown period would not be remarked unless its magnitude was such as to visibly affect the individual comparisons of theory with observations; and Hansen's Tables, as corrected, are the first ones of which the residual errors are so small that a term of $1''.5$ would be remarked in the comparison with observations.

On the Determination of the Distances of a Comet from the Earth from Three Observations. By Dr. F. W. Berg.

Let us designate by K_{II} the longitude of the ascending node, and by T_{II} the inclination to the ecliptic, of a great circle passing through the first and second observed places of the heavenly body, which may be given by means of $\alpha, \alpha', \beta, \beta'$, and we have—

$$\tan \beta = \sin (\alpha - K_{II}) \tan T_{II},$$

$$\tan \beta' = \sin (\alpha' - K_{II}) \tan T_{II};$$

and therefore, for the great circle passing through the second and third places, and for the great circle passing through the first and third places, we obtain—

$$\tan \beta' = \sin (\alpha' - K) \tan T,$$

$$\tan \beta'' = \sin (\alpha'' - K) \tan T,$$

$$\tan \beta = \sin (\alpha - K) \tan T,$$

$$\tan \beta'' = \sin (\alpha'' - K_{II}) \tan T_{II}.$$

Substituting these abbreviations in the Gaussian expressions (O.I.2.), (O.O.2), etc. (conf. *Theoria Motus*, art. 113), we find—

$$(O.I.2.) = \sin (\alpha'' - \alpha') . M, \quad (I.I.2.) = \sin (\alpha'' - \alpha') . M', \quad (II.I.2.) = \sin (\alpha'' - \alpha') . M'',$$

$$(O.O.2.) = \sin (\alpha - \alpha'') . M_{II}, \quad (O.I.2.) = \sin (\alpha - \alpha'') . M'_I, \quad (O.II.2.) = \sin (\alpha - \alpha'') . M''_I,$$

$$(O.I.O.) = \sin (\alpha' - \alpha) . M_{II}, \quad (O.I.I.) = \sin (\alpha' - \alpha) . M'_{II}, \quad (O.I.II.) = \sin (\alpha' - \alpha) . M''_{II},$$

if we put—

$$M = \tan B + \sin (K - L) \tan T,$$

$$M_I = \tan B + \sin (K_I - L) \tan T_I,$$

$$M_{II} = \tan B + \sin (K_{II} - L) \tan T_{II};$$

and these expressions will give $M', M'_I, M'_{II}, M'', M''_I, M''_{II}$, if we change B and L into $B' L'$ and $B'' L''$.

Therefore, we obtain from the equations (9), (10), (11) (conf. *Theoria Motus*, art. 114)—

$$(A) \quad \begin{aligned} 0 &= n (O.I.2) \delta + \sin (\alpha'' - \alpha') . P, \\ 0 &= n' (O.I.2) \delta' + \sin (\alpha'' - \alpha) . P_{II}, \\ 0 &= n'' (O.I.2) \delta'' + \sin (\alpha' - \alpha) . P_{II}, \end{aligned}$$

if we put

$$P = n M D - n' M' D' + n'' M'' D'',$$

$$P_I = n M_I D - n' M'_I D' + n'' M''_I D'',$$

$$P_{II} = n M_{II} D - n' M'_{II} D' + n'' M''_{II} D''.$$

For the case where the great circle through the first and third observed places of the body passes also through the second place, we have

$$(O.I.2) = 0,$$

and

$$T = T_I = T_{II}, \quad K = K_I = K_{II};$$

and, consequently,

$$\begin{aligned} M &= M, = M_{,,}, \\ M' &= M', = M_{,,}', \\ M'' &= M'', = M_{,,}'', \end{aligned}$$

and the equations (A) give only the relation

$$P = P, = P_{,,} = 0,$$

whence

$$(B) \quad nMD - n'M'D' + n''M''D'' = 0.$$

It is to be here remarked that the values $P, P, P_{,,}$ are of the third order with reference to the intervals of time.

In the formula (B) we may change n, n', n'' into N, N', N'' , where N, N', N'' have the same signification for the orbit of the Earth which have n, n', n'' for the orbit of the comet.

We hence obtain

$$(C) \quad NMD - N'M'D' + N''M''D'' = 0;$$

and, consequently, we obtain from (B) and (C)

$$\left(\frac{n}{n'} - \frac{N}{N'}\right) MD + \left(\frac{n''}{n'} - \frac{N''}{N'}\right) M''D'' = 0;$$

and therefore

$$(D) \quad n : n' : n'' = N : N' : N''. *$$

If we divide the third of the equations (A) by the first, we obtain in general:

$$(E) \quad \frac{\delta''}{\delta} = \frac{n}{n''} \cdot \frac{\sin(\alpha' - \alpha)}{\sin(\alpha'' - \alpha)} \cdot \frac{P_{,,}}{P};$$

and, consequently, in the case in which

$$(O.I.2) = 0,$$

we obtain

$$(F) \quad \frac{\delta''}{\delta} = \frac{n}{n''} \cdot \frac{\sin(\alpha' - \alpha)}{\sin(\alpha'' - \alpha)};$$

or, having regard to (D),

$$(G) \quad \frac{\delta''}{\delta} = \frac{N}{N''} \cdot \frac{\sin(\alpha' - \alpha)}{\sin(\alpha'' - \alpha)}.$$

The value of the fraction $\frac{P_{,,}}{P}$ is equal to unity only when the equation $(O.I.2) = 0$ is accurately true; but if the equation $(O.I.2)$

* Compare Gauss, *Theoria Motus Corporum Caelestium*, art. 162, and Olbers, *Abhandlung über die leichteste Methode die Bahn eines Cometes zu bestimmen*, herausg. von Encke, Weimar, 1847, p. 80.

$=0$ is only the result of errors of observation, the value of the fraction $\frac{P''}{P}$ may be affected with an error of the order zero with reference to the intervals of time, which error may be sometimes large.*

Wilna Observatory,
April 27 (May 9), 1876.

NOTE BY THE EDITOR.—In the foregoing paper the notation is as follows :

α , geocentric longitude of Comet,
 β , „ latitude,
 δ , „ distance, projected on plane of Ecliptic,
 L , heliocentric longitude of Earth,
 B , „ latitude,
 D , „ distance, projected on plane of Ecliptic,

viz., these values refer to the first observation; the corresponding accented quantities α' etc. refer to the second observation, and the doubly accented quantities α'' etc. to the third observation.

We have for the great circle passing through the second and third geocentric places of the Comet :

K , longitude of ascending node,
 T , inclination,

in reference to the Ecliptic; and similarly for great circle through third and first places K , T ; and for great circle through first and second places K , T .

The Gaussian function ($0, 1, 2$) denotes

$$\tan \beta \sin (\alpha'' - \alpha') + \tan \beta' \sin (\alpha - \alpha'') + \tan \beta'' \sin (\alpha' - \alpha);$$

and if herein for (α, β) , (α', β') , or (α'', β'') , we write (L, B) (L' , B') or (L'', B'') the symbol $0, 1$, or 2 , as the case may be, is changed into $0, 1$, or II .

ERRATA.

Page 306, second line from bottom, before the words "in the Dorpat," the words "*not seen*" should be inserted.

Page 307, seventeenth line from bottom, after the words "which shew" the word "*no*" should be inserted.

These errors reverse the meaning.

Page 308, fifth line from top, for *speculæ* read *specula*.

* Compare *Briefwechsel zwischen Olbers und Bessel*, herausg. von Erman, Leipzig, 1852, Band I. pp. 384 and 387.